SOLUTION OF EXERCISE # 5.3

Exercise # 5.3

In any triangle ABC if:

Q.1: $a = 10, \quad b = 15, \quad \beta = 50^\circ$, find α.

Sol. By using Law of Sines: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

We take $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$$\frac{10}{\sin\alpha} = \frac{15}{\sin 50^\circ}$$

$$\frac{\sin\alpha}{10} = \frac{\sin 50^\circ}{15}$$

$$\sin\alpha = \frac{10\sin 50^\circ}{15}$$

$$\sin\alpha = 0.5107$$

$$\alpha = \sin^{-1}(0.5107) \Rightarrow \boxed{\alpha = 30^\circ 42' 37''}$$

Q.2: $a = 20, \quad c = 32, \quad \gamma = 70^\circ$, find α.

Sol. By using Law of Sines: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

We take $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$$\frac{20}{\sin\alpha} = \frac{32}{\sin 70^\circ}$$

$$\frac{\sin\alpha}{20} = \frac{\sin 70^\circ}{32}$$

$$\sin\alpha = \frac{20\sin 70^\circ}{32}$$

$$\sin\alpha = 0.5873$$

$$\alpha = \sin^{-1}(0.5873) \Rightarrow \boxed{\alpha = 35^\circ 57' 56''}$$

Q.3: $a = 3, \quad b = 7, \quad \beta = 85^\circ$, find α.

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Sol. By using law of Sines: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

We take $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$$\frac{3}{\sin\alpha} = \frac{7}{\sin 85^\circ}$$

$$\frac{\sin\alpha}{3} = \frac{\sin 85^\circ}{7}$$

$$\sin\alpha = \frac{3\sin 85^\circ}{7}$$

$$\sin\alpha = 0.4269$$

$$\alpha = \sin^{-1}(0.4269) \Rightarrow \boxed{\alpha = 25^\circ 16' 24''}$$

Q.4: $a = 5$, $c = 6$, $\alpha = 45^\circ$, find γ.

Sol. By using law of sines: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

We take $\dfrac{c}{\sin\gamma} = \dfrac{a}{\sin\alpha}$

$$\frac{6}{\sin\gamma} = \frac{5}{\sin 45^\circ}$$

$$\frac{\sin\gamma}{6} = \frac{\sin 45^\circ}{5} \Rightarrow \sin\gamma = \frac{6\sin 45^\circ}{5}$$

$$\sin\gamma = 0.8485$$

$$\gamma = \sin^{-1}(0.8485) \Rightarrow \boxed{\gamma = 58^\circ 3' 6''}$$

Q.5: $a = 20\sqrt{3}$, $\alpha = 75^\circ$, $\gamma = 60^\circ$, find c (IA-2021)

Sol. By using law of sines: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

We take $\dfrac{c}{\sin\gamma} = \dfrac{a}{\sin\alpha}$

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$$\frac{c}{\sin 60^\circ} = \frac{20\sqrt{3}}{\sin 75^\circ}$$

$$c = \frac{(20\sqrt{3})\sin 60^\circ}{\sin 75^\circ} \Rightarrow \boxed{c = 31.06}$$

Q.6: $a = 211.3,\ \beta = 48^\circ 16',\ \gamma = 71^\circ 38'$ **find b.**

Sol. We know that: (IA-2019)

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha = 180^\circ - \beta - \gamma$$

$$\alpha = 180^\circ - 48^\circ 96' - 71^\circ 38'$$

$$\boxed{\alpha = 60^\circ 6'}$$

Now by using law of sines: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

We take $\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$

$$b = \frac{a\sin\beta}{\sin\alpha} = \frac{(211.3)\sin 48^\circ 16'}{\sin 60^\circ 6'} \Rightarrow \boxed{b = 181.89}$$

Q.7: $a = 18,\ \alpha = 47^\circ,\ \beta = 102^\circ$, **find c.**

Sol. We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 47^\circ - 102^\circ$$

$$\boxed{\gamma = 31^\circ}$$

Now by using law of sines: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

We take $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$

$$c = \frac{a\sin\gamma}{\sin\alpha} = \frac{18\sin 31^\circ}{\sin 47^\circ} \Rightarrow \boxed{c = 12.68}$$

Q.8: $a = 475,\ \beta = 72^\circ 15',\ \gamma = 43^\circ 30'$, **find b.**

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Sol. We know that

$$\alpha+\beta+\gamma=180^\circ$$
$$\alpha=180^\circ-\beta-\gamma$$
$$\alpha=180^\circ-72^\circ15'-43^\circ30' \Rightarrow \boxed{\alpha=64^\circ15'}$$

Now by using Law of Sines: $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}=\dfrac{c}{\sin\gamma}$

We take $\dfrac{b}{\sin\beta}=\dfrac{a}{\sin\alpha}$

$$b=\frac{a\sin\beta}{\sin\alpha}=\frac{475\sin 72^\circ15'}{\sin 64^\circ15'} \Rightarrow \boxed{b=502.26}$$

Q.9: $b=82$, $\beta=57^\circ$, $\gamma=78^\circ$, **find a.**

Sol. We know that

$$\alpha+\beta+\gamma=180^\circ$$
$$\alpha=180^\circ-\beta-\gamma$$
$$\alpha=180^\circ-57^\circ-78^\circ=\boxed{45^\circ}$$

Using law of sines: $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}=\dfrac{c}{\sin\gamma}$

We take $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}$

$$a=\frac{b\sin\alpha}{\sin\beta}=\frac{82\sin 45^\circ}{\sin 57^\circ} \Rightarrow \boxed{a=69.13}$$

Q.10: $\alpha=60^\circ$, $\beta=45^\circ$, **find the ratio of b to c. (IIA-2020)**

Sol. We know that

$$\alpha+\beta+\gamma=180^\circ$$
$$\gamma=180^\circ-\alpha-\beta$$
$$\gamma=180^\circ-60^\circ-45^\circ$$
$$\gamma=75^\circ$$

By using law of sines: $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}=\dfrac{c}{\sin\gamma}$

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We take $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

$$\frac{b}{c} = \frac{\sin\beta}{\sin\gamma}$$

$$\frac{b}{c} = \frac{\sin 45^\circ}{\sin 75^\circ}$$

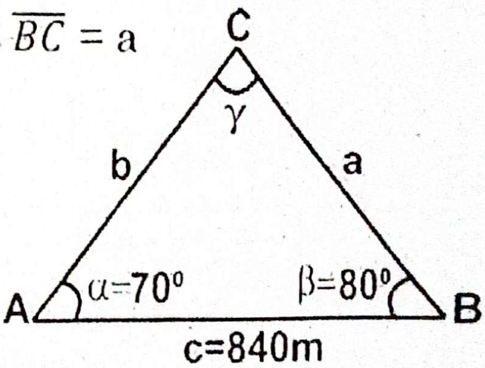
$$\frac{b}{c} = 0.7321 \quad \Rightarrow \quad \boxed{b : c = 0.7321}$$

Q.11: Two shore batteries at A & B, 840m apart are firing at a target C. The measure of angle ABC is 80º and the measure of angle BAC is 70º. Find the measures of distance AC and BC.

Sol. According to the statement, from figure:

$c = 840\text{m}, \quad \alpha = 70^\circ, \quad \beta = 80^\circ$

We will find $\overline{AC} = b$ and $\overline{BC} = a$



We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 70^\circ - 80^\circ$$

$$\gamma = 30^\circ$$

Now by using law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

We take $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

$$b = \frac{c\sin\beta}{\sin\gamma} = \frac{840\sin 80^\circ}{\sin 30^\circ}$$

$$\boxed{b = \overline{AC} = 1654.47\,\text{m}}$$

Again we take $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{840\sin 70^\circ}{\sin 30^\circ}$$

$$\boxed{a = \overline{BC} = 1578.68\,\text{m}}$$